Chapter 262

Balanced Incomplete Block Designs

Introduction

This module generates balanced incomplete block designs. Designs for up to ten treatments are available.

In order to make precise measurements of treatment means, uniform experimental conditions should be maintained when comparing a number of treatments. This ensures that differences among the treatment means result from the application of the treatment and not from some extraneous factor. To achieve this, experimental trials are often grouped together into blocks. In such designs, conditions are kept constant within the blocks and allowed to vary between the blocks. The best-known design of this type is the *randomized block* design. In this design, all treatments are present in each block.

Occasionally, the size of convenient blocks will not accommodate all the treatments of interest. For example, suppose you wanted to test four types of automobile tires for wear. An obvious choice for a block would be an automobile. You might select ten automobiles for the study. Assuming that the tires were rotated among the four positions, this experiment would control for differences in tire wear due to the type of automobile and the terrain that each traveled. However, what would you do if you wanted to test six types of tires. You could redesign the automobile, or you could adopt a *balanced incomplete block* design.

In a balanced incomplete block design, the treatments are assigned to the blocks so that every pair of treatments occurs together in a block the same number of times. This achieves the *balance* that is described in the title of the procedure. The balance means that all differences between treatments are measured with equal precision.

Following is an example of how four treatments are assigned to blocks with a natural size of three experimental units. Four blocks are required for this balanced incomplete block design.

Block	Treatment
1	A B C
2	A B D
3	A C D
4	B C D

Note that each treatment occurs three times in this experimental layout. Also note that each pair of treatments occurs twice. These are the basic properties of the balanced incomplete designs.

Box, Hunter, and Hunter (1978) point out the following rules when using such designs.

1. Randomly assign the numbers to the blocks.
2. Randomly assign the letters to the treatments.
3. Randomly assign the treatments within the blocks.
4. Randomly group blocks as replicates. A replicate is a complete set of all treatments.

If you take these steps, this design can be used effectively in those situations in which the block size and the number of treatments do not match.

Design Limits

These designs were taken from Cochran and Cox (1992). We have included designs with up to ten treatments. The following table shows what block sizes are available for each number of treatments.

Number of Treatments	Block Sizes Available
4	2, 3
5	2, 3, 4
6	2, 3, 4, 5
7	2, 3, 4, 6
8	2, 4, 7
9	2, 4, 5, 6, 8
10	2, 3, 4, 5, 6, 9

Note that some block sizes are not available for certain numbers of treatments.

Example 1 – Balanced Incomplete Block Design

This section presents an example of how to generate a balanced incomplete block design using this program. **CAUTION: since the purpose of this routine is to generate (not analyze) data, you should always begin with an empty dataset.**

In this example, we will show you how to generate a design with four treatments in blocks of two experimental units each.

Setup

To run this example, complete the following steps:

1 Specify the Balanced Incomplete Block Designs procedure options

- Find and open the **Balanced Incomplete Block Designs** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings template. To load this template, click **Open Example Template** in the Help Center or File menu.

Option	Value
Design Tab	
Block Size	**2**
Treatment Values	**1 2 3 4**
Store the Design Data in the Data Table	**Checked**

2 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

BIBD with Four Treatments in Blocks of Two

Experimental Design

Row	Block	Treatment
1	1	1
2	1	2
3	2	3
4	2	4
5	3	1
6	3	3
7	4	2
8	4	4
9	5	1
10	5	4
11	6	2
12	6	3

These values were also written to the Data Table.

Two columns are filled with data. The first column contains the block identification number. The second column contains the treatment number.

We note that this design calls for six blocks of two experimental units each.

To use this design, you would follow the randomization rules discussed earlier to obtain your experimental layout. After running your experiment, you would place the response values in column 3 (C3) from your experiment. You would then analyze the data using the GLM procedure following the instructions for the randomized block design. You would specify blocks as Random and treatment as Fixed. The response variable would be C3. On the Model window of the GLM ANOVA procedure, you would set Which Model Terms to “Up to 1-Way.” This forces the program to treat the block-by-treatment interaction as the error term.

Analysis of Variance Table

Expected Mean Squares Section

Source Term	DF	Term Fixed?	Denominator Term	Expected Mean Square
A: Block	5	No	S(AB)	S+bsA
B: Treatment	3	Yes	S(AB)	S+asB
S(AB)	3	No		S

Note: Expected Mean Squares are for the balanced cell-frequency case.

Analysis of Variance Table

Source Term	DF	Sum of Squares	Mean Square	F-Ratio	Prob Level	Power (Alpha=0.05)
A: Block	5	1895.75	379.15	8.23	0.056530	
B: Treatment	3	615.25	205.0833	4.45	0.125817	0.362064
S	3	138.25	46.08333			
Total (Adjusted)	11	2784.917				
Total	12					

* Term significant at alpha = 0.05

Since you are using random numbers for the response, the values of the sum of squares, mean squares, and F-ratios will not match those displayed here. However, the number of degrees of freedom will match.

Also note that the Expected Mean Square values are generated for a complete model. Since the balanced incomplete model is not complete, these values are incorrect.