

## Chapter 492

# Minimum Cost Capacitated Flow

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### Introduction

The *minimum cost capacitated flow* model is prominent among network flow models because so many other network models are special cases. The maximum flow, shortest-path, transportation, transshipment, and assignment models are all special cases of this model.

There are specialized algorithms that can be used to solve the minimum cost flow model. However, Hillier and Lieberman (2015) show that the model can be solved using linear programming. This is the approach we have taken in NCSS.

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### Data Structure

This procedure requires a special data format that includes two regions of the spreadsheet. These are the Arc Region which define the arcs, and the Node Region which defines the net flows at the nodes.

An Arc Region, defined using spreadsheet columns, gives the arcs (links, branches, or edges) of the network. Each row in this region represents an arc. The arc definition includes the name of starting node, ending node, incremental cost (cost of a single product), upper bound (if any), and lower bound (if any). The algorithm searches for the flows in the arcs. If flows can occur in both directions within an arc, two arcs must be included in the model.

A Node Region, defined using additional spreadsheet columns, gives the net flow through each node.

### Example Model

The following dataset shows how to prepare the example on page 399 of Hillier and Lieberman (2015) for analysis. Suppose a company produces a product at two factories. It can ship product among factories, from factory to a warehouse, or from factory to two stores. Although not all factories, stores, and the warehouse are connected, many are. These connections are defined using columns Node1 and Node2. The cost of shipping a single product unit are given in the IncCost column. There are no special lower bounds on shipment amounts within the arcs. However, only up to 10 units can be sent from Factory1 to Factory2 and only up to 80 units can be sent from the warehouse to Store1.

The supply and demand values are defined in the Node Region. The amount produced by each factory (the supply) are shown as positive net flows. Thus, Factory1 produces 50 units and Factory2 produces 40 units. The amounts needed at the two stores are 30 and 60 units. These are shown as negative numbers since these units are leaving the network. Since the warehouse does not consume any product, its net flow is zero.

Note that the sum of the net flows is exactly zero. This is a requirement of the model.

These data are stored in the dataset *Min Cost Flow*.

Minimum Cost Capacitated Flow

Min Cost Flow dataset

Node1	Node2	IncCost	UpperBnd	LowerBnd	Node	NetFlow
Factory1	Factory2	2	10	0	A: Factory1	50
Factory1	Warehouse	4		0	B: Factory2	40
Factory1	Store1	9		0	C: Warehouse	0
Factory2	Warehouse	3		0	D: Store1	-30
Warehouse	Store1	1	80	0	E: Store2	-60
Store1	Store2	3		0		
Store2	Store1	2		0		

## Linear Programming Formulation of the Minimum Cost Capacitated Flow Model

As stated earlier, we use a linear programming algorithm to solve for the maximum. Rather than present all the equations, we show how the above example is translated into a linear programming tableau. The LP tableau for the above data is

		X <sub>AB</sub>	X <sub>AC</sub>	X <sub>AD</sub>	X <sub>BC</sub>	X <sub>CE</sub>	X <sub>DE</sub>	X <sub>ED</sub>	RHS
Maximize	z =	2	4	9	3	1	3	2	
<b>Nodes</b>									
A		1	1	1					= 50
B		-1			1				= 40
C			-1		-1	1			= 0
D				-1			1	-1	= -30
E						-1	-1	1	= -60
Capacity		10				80			

## Example 1 – Minimum Cost Flow from Factory to Store

### Example Model

The following dataset shows how to prepare the example on page 399 of Hillier and Lieberman (2015) for analysis. Suppose a company produces a product at two factories. It can ship product among factories, from factory to a warehouse, or from factory to two stores. Although not all factories, stores, and the warehouse are connected, many are. These connections are defined using columns Node1 and Node2. The cost of shipping a single product unit are given in the IncCost column. There are no special lower bounds on shipment amounts within the arcs. However, only up to 10 units can be sent from Factory1 to Factory2 and only up to 80 units can be sent from the warehouse to Store1.

The supply and demand values are defined in the Node Region. The amount produced by each factory (the supply) are shown as positive net flows. Thus, Factor1 produces 50 units and Factory2 produces 40 units. The amounts needed at the two stores are 30 and 60 units. These are shown as negative numbers since these units are leaving the network. Since the warehouse does not consume any product, its net flow is zero.

Note that the sum of the net flows is exactly zero. This is a requirement of the model.

These data are stored in the dataset *Min Cost Flow*.

### Min Cost Flow dataset

Node1	Node2	IncCost	UpperBnd	LowerBnd	Node	NetFlow
Factory1	Factory2	2	10	0	A: Factory1	50
Factory1	Warehouse	4		0	B: Factory2	40
Factory1	Store1	9		0	C: Warehouse	0
Factory2	Warehouse	3		0	D: Store1	-30
Warehouse	Store1	1	80	0	E: Store2	-60
Store1	Store2	3		0		
Store2	Store1	2		0		

## Setup

To run this example, complete the following steps:

### 1 Open the Min Cost Flow example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Min Cost Flow** and click **OK**.

### 2 Specify the Minimum Cost Capacitated Flow procedure options

- Find and open the **Minimum Cost Capacitated Flow** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings template. To load this template, click **Open Example Template** in the Help Center or File menu.

<u>Option</u>	<u>Value</u>
<b>Specifications Tab</b>	
Arc Node 1 (Starting) Column .....	<b>Node1</b>
Arc Node 2 (Ending) Column.....	<b>Node2</b>
Arc Cost Column.....	<b>IncCost</b>
Arc Flow Capacity Column .....	<b>UpperBnd</b>
Node Name Column .....	<b>Node</b>
Node Net Flow Column .....	<b>NetFlow</b>

## Minimum Cost Capacitated Flow

## 3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

## Solution

## Solution

Arc Row	Solution Flow	IncCost	Total Cost	UpperBnd	Node1	Node2
2	40	4	160		Factory1	Warehouse
3	10	9	90		Factory1	Store1
4	40	3	120		Factory2	Warehouse
5	80	1	80	80	Warehouse	Store1
6	60	3	180		Store1	Store2
Total	230		630			

The solution is optimal.

This report shows the flows through each of the arcs. Arcs not listed receive a flow of zero. The flow through the network is 230 units at a cost of 630.

## Node Report

## Node Report

Type	Node	Flow In	Flow Out	Net Flow Out - In
Source	Factory1	0	50	50
Source	Factory2	0	40	40
Transshipment	Warehouse	80	80	0
Destination	Store1	90	60	-30
Destination	Store2	60	0	-60

This report shows the flows through each of the nodes. The *Flow In* is the sum of the flows of all arcs enter this node. The *Flow Out* is the sum of the flows of all arcs that leave this node.

## Model Network Arcs

## Model Network Arcs

Arc Row	IncCost	UpperBnd	Node1	Node2
1	2	10	Factory1 <sup>1</sup>	Factory2 <sup>1</sup>
2	4		Factory1 <sup>1</sup>	Warehouse
3	9		Factory1 <sup>1</sup>	Store1 <sup>2</sup>
4	3		Factory2 <sup>1</sup>	Warehouse
5	1	80	Warehouse	Store1 <sup>2</sup>
6	3		Store1 <sup>2</sup>	Store2 <sup>2</sup>
7	2		Store2 <sup>2</sup>	Store1 <sup>2</sup>

<sup>1</sup> Source (Supply) Node

<sup>2</sup> Destination (Demand) Node

This report lists all the arcs in the model as they were given on the dataset. The supply nodes are identified with a superscript of 1. The demand nodes are identified with the superscript of 2.