Chapter 435

Multidimensional Scaling

Introduction

Multidimensional scaling (MDS) is a technique that creates a map displaying the relative positions of a number of objects, given only a table of the distances between them. The map may consist of one, two, three, or even more dimensions. The program calculates either the metric or the non-metric solution. The table of distances is known as the *proximity* matrix. It arises either directly from experiments or indirectly as a correlation matrix.

To understand how the proximity matrix may be observed directly, consider the following marketing research example. Suppose ten subjects rate the similarities of six automobiles. That is, each subject rates the similarity of each of the fifteen possible pairs. The ratings are on a scale from 1 to 10, with "1" meaning that the cars are identical in every way and "10" meaning that the cars are as different as possible. The ratings are averaged across subjects, forming a similarity matrix. MDS provides the marketing researcher with a map (scatter plot) of the six cars that summarizes the results visually. This map shows the perceived differences between the cars.

The program offers two general methods for solving the MDS problem. The first is called *Metric*, or *Classical, Multidimensional Scaling (CMDS)* because it tries to reproduce the original metric or distances. The second method, called *Non-Metric Multidimensional Scaling (NMMDS)*, assumes that only the ranks of the distances are known. Hence, this method produces a map which tries to reproduce these ranks. The distances themselves are not reproduced.

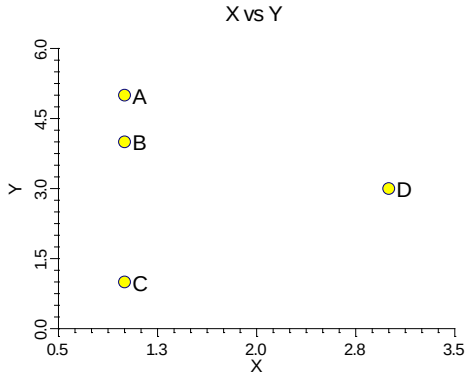
Discussion

The following example will help explain what MDS does. Consider the following set of data.

Original Data Matrix

Label	X	Y
A	1	5
B	1	4
C	1	1
D	3	3

A scatter plot of these data appears as follows:



Notice that the scatter plot lets us visually assess the distance between each pair of points. We can see that A is near B, but far from C and D. We can also see that C and D each seem to be by themselves. The actual distance between two points *i* and *j* may be computed numerically using the Euclidean distance formula:

$$d_{ij} = \sqrt{\sum_{k=1}^{p}\left(x_{ik} - x_{jk}\right)^2}$$

where p is the number of dimensions (which is 2 in our example), d_{ij} is the distance, and x_{ik} is the data value of the i^{th} row and k^{th} column. This formula is an simple extension of the famous Pythagorean Theorem. Note that this formula allows for an unlimited number of dimensions. That is, although we are only plotting the points in two-dimensional space, the formula computes the distance in p-dimensional space, where p can be greater than two.

For example, the distance from A to D is calculated as follows:

$$2.82843 = \sqrt{(1-3)^2 + (5-3)^2}\ 1$$

These distances are arranged in matrix format as follows:

Computed Distance Matrix

	A	B	C	D
A	0.00000	1.00000	4.00000	2.82843
B	1.00000	0.00000	3.00000	2.23607
C	4.00000	3.00000	0.00000	2.82843
D	2.82843	2.23607	2.82843	0.00000

Note that since the distance from A to D is the same as the distance from D to A, the distance matrix is symmetric. We only need to consider half of the matrix. In the program, we only require the upper half. The final distance matrix will be:

Upper-Triangular Distance Matrix

	A	B	C	D
A	0.00000	1.00000	4.00000	2.82843
B		0.00000	3.00000	2.23607
C			0.00000	2.82843
D				0.00000

The task attempted by MDS is that given only a distance matrix, find the original data so that a map (scatter plot) of the data may be drawn.

Some of the difficulties facing MDS may be seen even in this simple example. First, as the number of objects increases, the possible number of dimensions increases as well. If you have three objects, these will at most define a

two-dimensional plane. With four objects, you will usually find a three-dimensional space. And so on, with each new object adding one more possible dimension.

Also, notice that if the data are shifted in such a way that their positions relative to each other are maintained (rotated, translated, or transposed), the computed distance matrix will be the same. Hence, the distance matrix could have come from numerous sets of data.

A third challenge comes when the distances themselves are not actually known. You might only be given knowledge of their relative size.

MDS techniques have proved useful because circumstances often occur where the actual coordinates of the objects are not known, but some type of distance matrix is available. This is especially the case in psychology where people cannot draw an overall picture of a group of objects, but they can express how different individual pairs of objects are. From these pair-wise differences MDS often can provide a useful picture.

Goodness-of-Fit

As in any data analysis problem, an expression is needed to express how well a particular set of data are represented by the model that the analysis imposes. In the case of MDS, you are trying to model the distances. Hence, the most obvious choice for a goodness-of-fit statistic is one based on the differences between the actual distances and their predicted values. Such a measure is called *stress* and is calculated as values:

$$stress = \sqrt{\frac{\sum\left(d_{ij} - \hat{d}_{ij}\right)^2}{\sum d_{ij}^{\ 2}}}$$

Here $\hat{d}_{ij}$ is predicted distance based on the MDS model. Note that this predicted value depends on the number of dimensions kept and the algorithm that you used (metric versus non-metric).

As you can see from this equation, MDS fits with stress values near zero are the best.

In his original paper on MDS, Kruskal (1964) gave following advise about stress values based on his experience:

Stress	Goodness-of-fit
0.200	poor
0.100	fair
0.050	good
0.025	excellent
0.000	perfect

More recent articles caution against using a table like this since acceptable values of stress depends on the quality of the distance matrix and the number of objects in that matrix.

Number of Dimensions

One of the main tasks the analyst has is determining the number of dimensions in the MDS model. Each dimension represents a different underlying factor. One of the goals of the MDS analysis is to keep the number of dimensions as small as possible. Usually, the analyst will anticipate select two or, at most, three dimensions. If more are required, you may decide that MDS is not appropriate for your data.

The usual technique is to solve the MDS problem for a number of dimension values and adopt the smallest number of dimensions that achieves a reasonably small value of stress. The program displays a simple bar chart of the stress values to aid in the selection of the number of dimensions.

Some researchers also consider the relative size of the eigenvalues that are generated during the solution process. These eigenvalues are then used to determine the number of dimensions just as they are used in factor analysis to determine the number of factors.

Proximity Measures

Proximity measures quantify how "close" two objects are. The program accepts three forms of proximity values: dissimilarities, similarities, and correlations.

Dissimilarities represent the distance between two objects. They may be measured directly, as in the distance between two towns, or approximated, as in "Bill is five points different from Joe on a ten-point scale." MDS algorithms use the dissimilarities directly. A dissimilarity matrix is symmetrical.

Similarities represent how close (in some sense) two objects are. The program lets you enter a similarity measure for each pair of objects. Similarities must obey the rule: $similarity_{ij}$ <= $similarity_{ii}$ and $similarity_{jj}$ for all *i* and *j*. Similarity matrices are symmetrical.

Similarities are converted to dissimilarities using the formula:

$$d_{ij} = \sqrt{s_{ii} + s_{jj} - 2s_{ij}}$$

where d_{ij} represents a dissimilarity and s_{ij} represents a similarity.

When your data consists of standard measures rather than dissimilarities or similarities, you can create a dissimilarity matrix by first creating the correlation matrix and then using the above formula to convert the correlations to dissimilarities. The program automatically calculates pair-wise correlations for the variable you specify.

Comparison of Metric and Non-Metric MDS

Although the computations are simpler for the metric method than for the non-metric method, both seem to yield similar results when applied to well-known examples. When you have true distance data, the classical method yields a solution that can be used directly. When you only have dissimilarities, the non-metric approach is somewhat more appealing.

Metric MDS

Classical MDS procedures stem back to Torgerson (1952), who was one of the pioneers of the technique. His algorithm is explained next.

Suppose a distance matrix ***D*** approximates the inter-point distances of a configuration of points ***X*** in a space of low dimensionality *p* (usually *p* = 1, 2, or 3). That is, the elements of **D**, denoted d_{ij}, may be calculated from ***X*** using the following formula:

$$d_{ij} = \sqrt{\sum_{k=1}^{p} \left(x_{ik} - x_{jk}\right)^2}$$

The steps in the classical MDS algorithm are as follows:

1. From ***D*** calculate ***A*** = $\left\{-\frac{1}{2}d_{ij}^{\,2}\right\}$.
2. From ***A*** calculate ***B*** = $\left\{a_{ij} - a_{i.} - a_{.j} + a_{..}\right\}$, where $a_{i.}$ is the average of all a_{ij} across *j*.
3. Find the *p* largest eigenvalues $\lambda_1 > \lambda_2 > \cdots > \lambda_p$ of ***B*** and corresponding eigenvectors $L = \left(L_{(1)}, L_{(2)}, \cdots L_{(p)}\right)$ which are normalized so that $L'_{(i)}L_{(i)} = \lambda_i$. (We are assuming that *p* is selected so that the eigenvalues are all relatively large and positive.)
4. The coordinates of the objects are the *rows* of ***L***.

The classical solution is optimal in the least-squares sense. That is, when a direct solution is possible (i.e., when ***D*** is truly a Euclidean distance matrix), the solution, ***L,*** minimizes the sum of squared differences between the actual d_{ij}'s (elements of ***D***) and the $\hat{d}_{ij}$'s based on ***L***. Another way of saving this is that it minimizes the value of *stress*, where *stress* was defined above.

Non-Metric MDS

Implicit in the above is the assumption that there is a true configuration in *p* dimensions, i.e., that ***D*** is a distance matrix. Often, however, it is more realistic to assume a less stringent relationship between the observed distances (or dissimilarities) d_{ij} and the true distances, denoted δ_{ij}. That is, suppose we assume that

$$d_{ij} = f\left(\delta_{ij} + e_{ij}\right)$$

where e_{ij} represents errors of measurements, distortions, etc. Also, we assume that *f(x)* is an unknown, monotonically increasing function.

For this model, the only information we can use is the rank order of the d_{ij}. Usually, this approach is used when ***D*** is simply a dissimilarity matrix rather than a true distance matrix. This assumption is often more plausible in practical situations.

An algorithm to produce a solution based only on the rank order information was provided by Kruskal (1964). It is involved, so we will not reproduce it here. We note that Kruskal's algorithm minimizes stress.

Kruskal's algorithm uses steepest descent to find a local minimum from a given starting configuration. The choice of the starting configuration is important to finding the global rather than a local minimum. Many authors recommend using the solution of the metric MDS as the starting configuration. This is the default starting configuration in this program. You may also select several random starting configurations and compare the resulting stress values.

Data Structure

The data are may be entered in three formats. The first format is the standard row-column format from which the correlations have be calculated. The MDS conducted on the correlations in an attempt to determine which of the variables are similar. The second format is the upper-triangular portion of a distance matrix. The third format is the upper-triangular portion of a similarity matrix.

An example of an upper-triangular distance matrix is contained in the MDS2 database. We suggest that you open this database now so that you can follow along with the example.

MDS2 dataset

Sport	Hockey	Football	Basketball	Tennis	Golf	Croquet
Hockey	0	2	3	4	5	5
Football		0	3	5	6	5
Basketball			0	5	4	6
Tennis				0	4	3
Golf					0	2
Croquet						0

Example 1 – Metric Multidimensional Scaling

This section presents an example of how to run an analysis of the data contained in the MDS2 dataset.

Setup

To run this example, complete the following steps:

1 Open the MDS2 example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **MDS2** and click **OK**.

2 Specify the Multidimensional Scaling procedure options

- Find and open the **Multidimensional Scaling** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings template. To load this template, click **Open Example Template** in the Help Center or File menu.

Option	Value
Variables Tab	
Input Variables	**Hockey-Croquet**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Eigenvalue Section

Eigenvalue Section

Dim No.	Eigenvalue	Individual Percent	Cumulative Percent	Bar Chart
1	30.73	54.28	54.28	||||||||||||||||||||||||
2 (Used)	12.85	22.69	76.97	||||||||||
3	6.38	11.27	88.24	||||||
4	1.68	2.97	91.21	||
5	0.00	0.00	91.21	|
6	-4.98	8.79	100.00	|||||
Total	56.62			

This report is produced by CMDS.

In this particular example, the first two dimensions account for 77% of the variation while the first three dimensions account for 88%. We would probably use two or perhaps three dimensions.

Eigenvalues

These are the eigenvalues found during CMDS. The eigenvalues are helpful in determining the number of dimensions that are necessary to represent the dissimilarity matrix accurately. As in factor analysis, the task is to select enough dimensions to approximate the data, but few enough to keep the interpretation simple. The eigenvalue report allows you to quickly determine the impact of each new dimension.

In MDS, some of the eigenvalues can be negative. Do not keep these dimensions. The basic rule is to find the number of relatively large, positive eigenvalues. This report provides a bar graph and percentages to help you determine the number of dimensions.

Individual and Cumulative Percents

The first column gives the percentage of the total of the absolute value of the eigenvalues accounted for by this dimension. The second column is the cumulative total of the percentage.

Bar Chart

This is a rough bar plot of the eigenvalues. It enables you to quickly note the relative size of each eigenvalue. Many authors recommend it as a method of determining how many dimensions to retain.

Fit Summary Section

Fit Summary Section

No. Dim's	Squared Differences	Stress	Pseudo R-Squared
1	37.105982	0.364035	0.00
2	6.947666	0.157522	70.73
3	2.413305	0.092838	89.83
4	2.468686	0.093897	89.60

Number of Dissimilarities	15
Mean of Dissimilarities	4.133333
Sum of Squared Dissimilarities	280.000000
Mean Corrected Sum of Squared Dissimilarities	23.733333

This report provides information useful in determining the number of dimensions that are necessary and assessing the goodness-of-fit of the CMDS model.

No. Dim's

The number of dimensions used in calculating this row of statistics.

Squared Differences

The sum of the squared differences between the actual dissimilarity values and those predicted by the solution.

Stress

This is the value of the stress goodness-of-fit statistic. It is equal to the square root of the Squared Differences divided by the square root of the Sum of the Squared Dissimilarities. It is one of the most popular measures of accuracy of the fit. A value below 0.05 is acceptable. A value below 0.01 is considered good.

Pseudo R-Squared

This is an index, similar to the R-squared value in regression analysis, which indicates what percentage of the sum of squared dissimilarities (corrected for the mean) is accounted for by this number of dimensions. A value above 80% is hoped for.

Number of Dissimilarities

This is the number of dissimilarity values.

Mean of Dissimilarities

This is the mean of the dissimilarity values

Sum of Squared Dissimilarities

This is the sum of the squared dissimilarities. It is the denominator of the stress statistic.

Mean Corrected Sum of Squared Dissimilarities

This is the sum of the squared dissimilarities about their mean. It is the denominator of the Pseudo R-Squared statistic.

Solution Section

Solution Section

Variables	Dim1	Dim2	Dim3	Dim4
Hockey	1.9301	-0.6756	0.3818	1.0441
Football	2.6179	-1.1281	-1.1303	-0.4680
Basketball	2.1119	2.0914	0.4168	-0.4032
Tennis	-1.4786	-1.3608	1.8070	-0.3940
Golf	-2.3836	2.0059	-0.2743	0.2351
Croquet	-2.7976	-0.9328	-1.2011	-0.0140

This report presents the solution of the MDS procedure. These are the data that are plotted in the MDS map. They have been scaled so that the sum of squares for each column is equal to the eigenvalue for that dimension.

Note that these data were constructed so that the distance between two rows is close to the original dissimilarity value.

Although some interpretation of these numbers may be made directly, usually the data are displayed on scatter plots.

Dissimilarity Section

Dissimilarity Section

Row	Column	Actual Dissimilarity	Predicted Dissimilarity	Actual Difference	Percent Difference
1 Hockey	2 Football	2.000000	0.823324	1.176676	58.83
5 Golf	6 Croquet	2.000000	2.967759	-0.967759	-48.39
1 Hockey	3 Basketball	3.000000	2.772988	0.227012	7.57
2 Football	3 Basketball	3.000000	3.259039	-0.259039	-8.63
4 Tennis	6 Croquet	3.000000	1.386718	1.613282	53.78
1 Hockey	4 Tennis	4.000000	3.476854	0.523146	13.08
3 Basketball	5 Golf	4.000000	4.496329	-0.496329	-12.41
4 Tennis	5 Golf	4.000000	3.486229	0.513771	12.84
1 Hockey	5 Golf	5.000000	5.079231	-0.079231	-1.58
1 Hockey	6 Croquet	5.000000	4.734691	0.265309	5.31
2 Football	4 Tennis	5.000000	4.103106	0.896894	17.94
2 Football	6 Croquet	5.000000	5.419049	-0.419049	-8.38
3 Basketball	4 Tennis	5.000000	4.980893	0.019107	0.38
2 Football	5 Golf	6.000000	5.902321	0.097679	1.63
3 Basketball	6 Croquet	6.000000	5.766223	0.233777	3.90

Dimensions	2
Sum of Squared Dissimilarities	280.000000
Sum of Squared Differences	6.947666
Stress	0.157522
Pseudo R-Squared	70.726127

You might think of this as a residual analysis report since it highlights the differences between the actual and the predicted dissimilarities. It will let you focus on those dissimilarities that are not fit well by the model.

Row

The variable associated with this row of the dissimilarity matrix.

Column

The variable associated with this column of the dissimilarity matrix.

Actual Dissimilarity

The value from the input (or calculated) dissimilarity matrix for this row and column.

Predicted Dissimilarity

The predicted dissimilarity value based on the number of dimensions that you have selected.

Actual Difference

The Actual Dissimilarity minus the Predicted Dissimilarity. This value shows the size of the error in predicting this element of the dissimilarity matrix.

Percent Difference

The percentage the Actual Difference is of the Actual Dissimilarity. This value highlights the outliers--those dissimilarities that are not fit well by the MDS model.

Dimensions

The number of dimensions used in calculating the statistics.

Sum of Squared Dissimilarities

This is the sum of the squared dissimilarities. It is the denominator of the stress statistic.

Sum of Squared Differences

This is the sum of the squared differences. It is the numerator of the stress statistic.

Stress

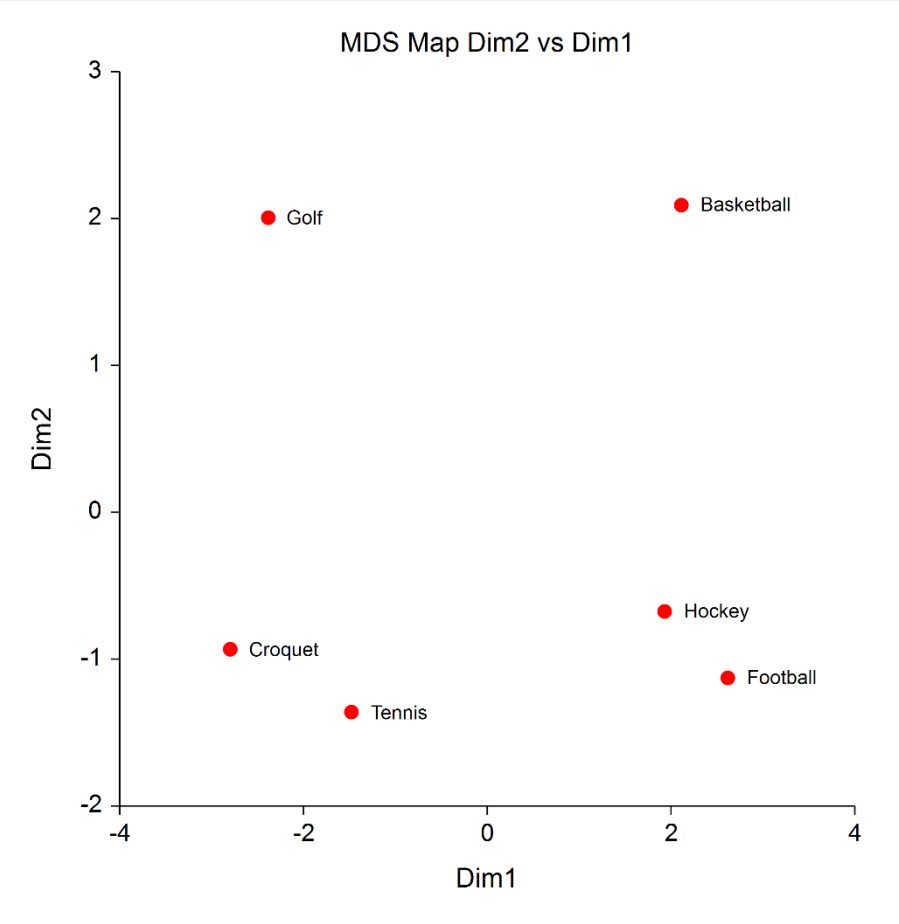
This is the value of the stress goodness-of-fit statistic. It is equal to the Squared Differences divided by the Sum of the Squared Dissimilarities. It is one of the most popular measures of accuracy of the fit. A value below 0.05 is acceptable. A value below 0.01 is considered good.

Pseudo R-Squared

This is an index, similar to the R-squared value in regression analysis, which indicates what percentage of the sum of squared dissimilarities (corrected for the mean) is accounted for by this number of dimensions. A value above 80% is hoped for.

MDS Map

MDS Map



This plot is the chief objective of an MDS analysis. It is often referred to as the MDS *map*. It allows you to interpret the dissimilarity matrix on a two-dimensional scatter plot.

There is no real orientation to this map. You could legitimately rotate the values around the plot's center. The main characteristics of interest are the relative positions of the points and any clusters that are apparent.

In this example, we see that the respondents considered hockey and football to be similar. They also considered croquet and tennis to be quite similar. Football appears quite different from golf. And so on. Notice how easy it is to draw conclusions about the similarities among the sports.

A second task of the MDS analyst is to find the underlying factors that respondents used when they created these dissimilarities. For example, a vertical line down the center of the plot would divide team sports on the right from individual sports on the left. We would hypothesize this as one interpretation of the Dim1 (horizontal) axis.

Example 2 – Non-Metric Multidimensional Scaling

This section presents an example of how to run an analysis of the data contained in the MDS2 dataset using NMMDS.

Setup

To run this example, complete the following steps:

1 Open the MDS2 example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **MDS2** and click **OK**.

2 Specify the Multidimensional Scaling procedure options

- Find and open the **Multidimensional Scaling** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 2** settings template. To load this template, click **Open Example Template** in the Help Center or File menu.

Option	Value
Variables Tab	
Input Variables	**Hockey to Croquet** ("Hockey-Croquet" will appear in the Input Variables box.)
Solution Type	**Non-Metric**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Eigenvalue Section

Eigenvalue Section

Dim No.	Eigenvalue	Individual Percent	Cumulative Percent	Bar Chart
1	30.73	54.28	54.28	||||||||||||||||||||||||
2 (Used)	12.85	22.69	76.97	|||||||||||
3	6.38	11.27	88.24	||||||
4	1.68	2.97	91.21	||
5	0.00	0.00	91.21	|
6	-4.98	8.79	100.00	|||||
Total	56.62			

This report is produced by CMDS which was used as the starting configuration. Its definitions were given above and will not be repeated here.

Non-Metric Iteration Summary Section

Non-Metric Iteration Summary Section

<table>
<tr><th>No.
Dim's</th><th>Percent Rank
Maintained</th><th>Stress</th><th>Why
Terminated</th><th>Bar Chart
of Stress</th></tr>
<tr><td>1</td><td>50.00</td><td>0.212628</td><td>Stress Change</td><td>|IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII<<I</td></tr>
<tr><td>2</td><td>42.86</td><td>0.052276</td><td>Stress Change</td><td>|IIIIIIIIIIIIIIIIIIIIIIIIII</td></tr>
<tr><td>3</td><td>64.29</td><td>0.000315</td><td>Stress Change</td><td>|</td></tr>
<tr><td>4</td><td>78.57</td><td>0.000006</td><td>Min Stress</td><td>|</td></tr>
</table>

This report provides information about the number of dimensions that are necessary and the goodness-of-fit of the solution.

No. Dim's

The number of dimensions used in calculating this row of statistics.

Percent Rank Maintained

The non-metric solution tries to maintain the rank ordering of the dissimilarities. This is the percentage of the dissimilarities whose rank order was maintained. The higher this value is, the better the quality of the solution.

Stress

Defined earlier, this is one of the most popular measures of accuracy of the fit. A value below 0.05 is acceptable. A value below 0.01 is considered good.

Why Terminated

This field explains which stopping rule caused the iterative procedure to stop. This is important to watch since the solution is not optimal if the maximum iterations were reached before the algorithm converged. When this happens, you should change some of the iteration control parameters, especially the number of iterations.

Bar Chart of Stress

This column graphically portrays the stress values. You want to choose the fewest number of dimensions that give you a small stress value.

Solution Section

Solution Section

Variables	Dim1	Dim2
Hockey	0.3306	-0.1495
Football	0.4234	-0.0227
Basketball	0.2674	0.2837
Tennis	-0.2542	-0.3538
Golf	-0.3473	0.2227
Croquet	-0.4199	0.0196

This report presents the final configuration of the NMMDS procedure. These are the data that are plotted in the MDS map.

Note that these data were not constructed so that the distance between two rows is close to the original dissimilarity value. Instead, the non-metric solution attempts to maintain the same rank ordering of the calculated distances as occur in the original dissimilarity matrix. Although some interpretation of these numbers may be made directly, usually the data are displayed on scatter plots.

Dissimilarity Section

Dissimilarity Section

Row	Column	Actual Dissimilarity	Predicted Dissimilarity
1 Hockey	2 Football	2.000000	0.157123
5 Golf	6 Croquet	2.000000	0.215646
1 Hockey	3 Basketball	3.000000	0.437798
2 Football	3 Basketball	3.000000	0.343893
4 Tennis	6 Croquet	3.000000	0.408445
1 Hockey	4 Tennis	4.000000	0.619391
3 Basketball	5 Golf	4.000000	0.617695
4 Tennis	5 Golf	4.000000	0.583869
1 Hockey	5 Golf	5.000000	0.773283
1 Hockey	6 Croquet	5.000000	0.769237
2 Football	4 Tennis	5.000000	0.754122
2 Football	6 Croquet	5.000000	0.844336
3 Basketball	4 Tennis	5.000000	0.823650
2 Football	5 Golf	6.000000	0.808819
3 Basketball	6 Croquet	6.000000	0.736258

You might think of this as a residual analysis report since it highlights the differences between the actual and the predicted dissimilarities. It will let you focus on those dissimilarities that are not fit well by the model.

This report presents the details of how well the rank ordering of the dissimilarity values is preserved in the final configuration. Note that the predicted values are quite different from the actual values since all the algorithm was attempting to do was maintain the ordering.

Row

The variable associated with this row of the dissimilarity matrix.

Column

The variable associated with this column of the dissimilarity matrix.

Actual Dissimilarity

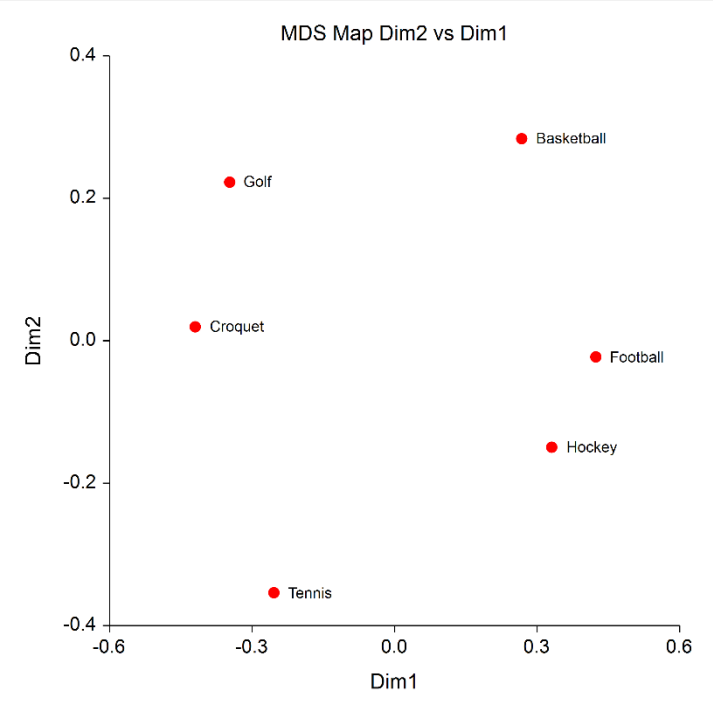
The value from the input (or calculated) dissimilarity matrix for this row and column.

Predicted Dissimilarity

The predicted dissimilarity value based on the number of dimensions that you have selected. Note that this is not predicting the actual dissimilarity value, but some unknown function of the dissimilarity value. It is not usually necessary to determine the function. We are mainly interested in how well the ordering of the actual values is maintained by these predicted values.

MDS Map

MDS Map



This plot is the chief objective of an MDS analysis. It is often referred to as the MDS *map*. It allows you to interpret the dissimilarity matrix on a two-dimensional scatter plot.

There is no real orientation to this map. You could legitimately rotate the values around the plot's center. The main characteristics of interest are the relative positions of the points and any clusters that are apparent.

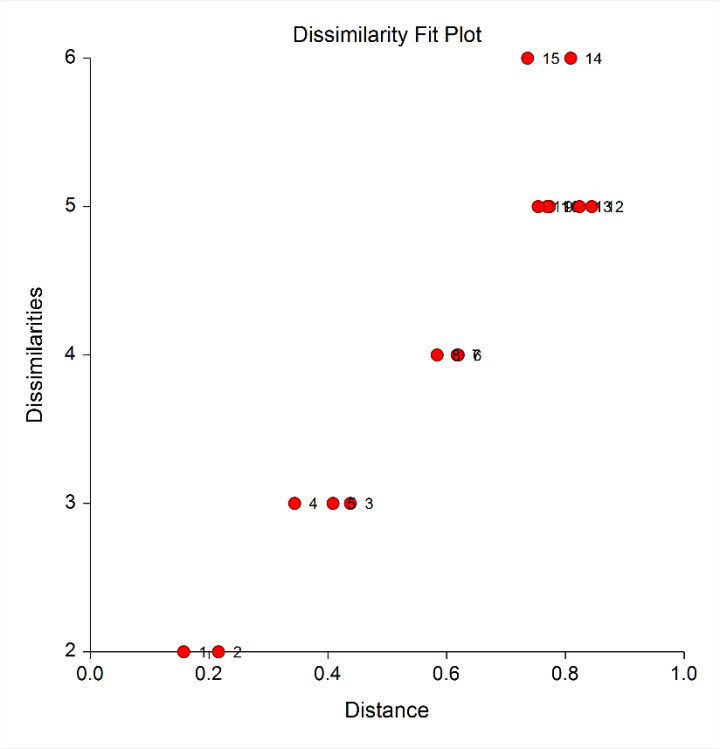
In this example, we see that the respondents considered hockey and football to be similar. They also considered croquet and golf to be similar. Football appears quite different from croquet. And so on. Notice how easy it is to draw conclusions about the similarities among the sports.

A second task of the MDS analyst is to find the underlying factors that respondents used when they created these dissimilarities. For example, a vertical line down the center of the plot would divide team sports on the right from individual sports on the left. We might hypothesize this as one interpretation of the Dim1 (horizontal) axis.

It is interesting to compare this map with the map produced by the metric solution. The main difference appears to be that golf and croquet are now much closer together (as they were rated in the original data). Again, football and basketball appear to be closer together in this plot as we might expect from the original data. In this case, the NMMDS map appears to be more accurate than the CMDS map. This is as we might expect since, the NMMDS procedure refined the CMDS map.

Dissimilarity Fit Plot

Dissimilarity Fit Plot



This graph plots the dissimilarity values on the vertical axis against the predicted dissimilarity values on the horizontal axis. The caliber of the solution depends upon this plot showing an upward-sloping trend. If the solution was perfect, then as you move across the plot from left to right, you would never go down from one point to the next.

We notice in this case that the solution confuses the large distances. This may be due to the large number of ties in this area (look at the Dissimilarity Section to see all the 5's and 6's).