Chapter 314

Robust Linear Regression (Passing-Bablok Median-Slope)

Introduction

This procedure performs robust linear regression estimation using the Passing-Bablok (1988) median-slope algorithm. Their original algorithm (1983, 1984) was designed for method comparisons in which it was desired to test whether the intercept is zero and the slope is one. This procedure extends that algorithm for the case where the main interest is in estimating the intercept and slope in the linear equation

$$Y = \beta_0 + \beta_1 X.$$

The estimate of the slope ($B1$) is calculated as the median of all slopes that can be formed from all possible pairs of data points, except those pairs that result in a slope of 0/0. To correct for estimation bias, the median is *shifted* by a factor *K* which is one-half the number of slopes that are less than zero. This creates an approximately unbiased estimator.

The estimate of the intercept ($B0$) is the median of $\{Y_i - B1\ X_i\}$.

This procedure is used for *transference*: when you want to rescale one reference interval to another scale.

Experimental Design

Typical designs suitable for Passing-Bablok regression include up to 3000 paired measurements, (x_i, y_i), $i = 1, \ldots ,$ n, similar to the common input for simple linear regression. Typical data of this type are shown in the table below.

Typical Data for Passing-Bablok Median-Slope Regression

Subject	X	Y
1	7	7.9
2	8.3	8.2
3	10.5	9.6
4	9	9
5	5.1	6.5
6	8.2	7.3
7	10.2	10.2
8	10.3	10.6
9	7.1	6.3
10	5.9	5.2

Technical Details

The methods and results in this chapter are based on the formulas given in Passing and Bablok (1983, 1984) and Bablok, Passing, Bender, and Schneider (1988).

Assumptions

Passing-Bablok regression requires the following assumptions:

1. The relationship between *X* and *Y* is linear (straight-line).
2. No special assumptions are made about the distributions (including the variances) of *X* and *Y*.

Passing-Bablok Estimation

Define X_i and Y_i, *i* = 1, … , *n*, as the values for two variables, each sampled with error to give the observed values x_i and y_i, respectively. For each of the $N = \binom{n}{2}$ possible pairs of points, define the slope by

$$S_{ij} = \frac{y_i - y_j}{x_i - x_j}$$

with the following substitutions

If $x_i = x_j$ and $y_i = y_j$, the result is 0/0 which is undefined. Omit these pairs from the calculation of the slope.

If $x_i = x_j$ and $y_i > y_j$, $S_{ij} = +\infty$.

If $x_i = x_j$ and $y_i < y_j$, $S_{ij} = -\infty$.

If $x_i > x_j$ and $y_i = y_j$, $S_{ij} = +0$.

If $x_i < x_j$ and $y_i = y_j$, $S_{ij} = -0$.

The value of *K* is computed as [*neg*/2] where *neg* is the number of S_{ij}'s that are negative (including -0's and -∞'s).

The slope *B*1 is the shifted median of *Sij*, where the median is shifted to the right *K* steps.

Using this slope, calculate intercept *B*0 as the median of all *n* of the quantities $y_i - B1x_i$.

Confidence Bounds

Calculate the confidence bounds for β_1 as follows. Let $z_{\alpha/2}$ be the 1 – α/2 quantile of the standard normal distribution. Let

$$C_{\alpha/2} = z_{\alpha/2}\sqrt{\frac{n(n-1)(2n+5)}{18}}$$

Now calculate

$$M_1 = \left[\frac{N - C_{\alpha/2}}{2}\right]$$

Here, [U] rounds U to the nearest integer.

Also calculate

$$M_2 = N - M_1 + 1$$

Finally, a confidence interval for β_1 is given by

$$S_{(M_1+K)} \leq \beta_1 \leq S_{(M_2+K)}$$

Where the *Sij's* are sorted.

Confidence limits for the intercept are calculated as follows. Let $B1_L$ and $B1_U$ be the lower and upper limits for the slope from the last calculation. Now calculate the limits for the intercept as

$$B0_L = \text{median}\{y_i - B1_U x_i\}$$

and

$$B0_U = \text{median}\{y_i - B1_L x_i\}$$

Negative Kendall's Tau

If the data exhibit a negative value of Kendall's tau, the substitution $w = -y$ is used was switch the correlation to a positive value. Once estimation is complete, the estimate of $B1$ is set to $-B1_w$.

Kendall's Tau Test of the High Correlation Assumption

Passing and Bablok (1983) recommended that a preliminary two-sided test be conducted to determine if Kendall's tau correlation between X and Y is significantly different from zero. They also indicate that this correlation must be positive. Kendall's tau correlation is well documented in the *Correlation* procedure.

Example 1 – Passing-Bablok Median-Slope Regression

This section presents an example of how to run a Passing-Bablok regression analysis of the data in the *PassBablok1* dataset. This dataset contains measurements from two measurement methods on each of 30 items. The goal is to find robust estimates of the coefficients in the linear regression equation.

Setup

To run this example, complete the following steps:

1 Open the PassBablok 1 example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **PassBablok 1** and click **OK**.

2 Specify the Robust Linear Regression (Passing-Bablok Median-Slope) procedure options

- Find and open the **Robust Linear Regression (Passing-Bablok Median-Slope)** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings template. To load this template, click **Open Example Template** in the Help Center or File menu.

Option	Value
Variables Tab	
Y Variable	**Method3**
X Variable	**Method1**
Reports Tab	
Residuals	**Checked**
Plots Tab	
Passing-Bablok Regression Scatter Plot	**Checked**
Residual Plot	**Checked**

3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Run Summary Report

Run Summary

Item	Value	Item	Value
Y Variable	Method3	Rows Used	30
X Variable	Method1	B0: Intercept	99.9907
		B1: Slope	-0.9983

This report gives a summary of the input and various descriptive measures about the Passing-Bablok regression.

Descriptive Statistics Report

Descriptive Statistics

Item	Y	X
Variable	Method3	Method1
N	30	30
Mean	37.00667	63.14
Std Dev	23.5886	23.6153
Std Error	4.3067	4.3115
COV	0.6374	0.3740
Minimum	1.4	14.2
First Quartile	16.925	48.8
Median	35.85	61.4
Third Quartile	52.575	84.1
Maximum	87.3	99.1

This report gives descriptive statistics about the variables used in the regression.

Kendall’s Tau Correlation Confidence Interval and Hypothesis Test

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Kendall's Tau Correlation	Lower 95%: Conf. Limit of Tau	Upper 95%: Conf. Limit of Tau	Z-Value for Testing H0: ρ = 0	P-Value	Reject H0 that ρ = 0 at α = 0.05?
-0.9724	-0.9833	-0.9546	-7.5289	0.0000	Yes

This procedure assumes that Kendall's Tau is significantly different from zero.
This assumption cannot be rejected.

The section reports an analysis of Kendall’s tau correlation. It provides both a confidence interval and a significance test. The main thing to look for here is that the absolute correlation (-0.9724 in this example) is large. This can be surmised from both the confidence interval and the p-value.

Regression Coefficient

Regression Coefficients

Item	Intercept B0	Slope B1
Regression Coefficient	99.9907	-0.9983
Lower 95% Conf. Limit of β(i)	99.1920	-1.0098
Upper 95% Conf. Limit of β(i)	100.7432	-0.9854

This section reports the regression coefficients, along with their analytic confidence limits. **These estimates are the main focus of the analysis**.

Residuals Report

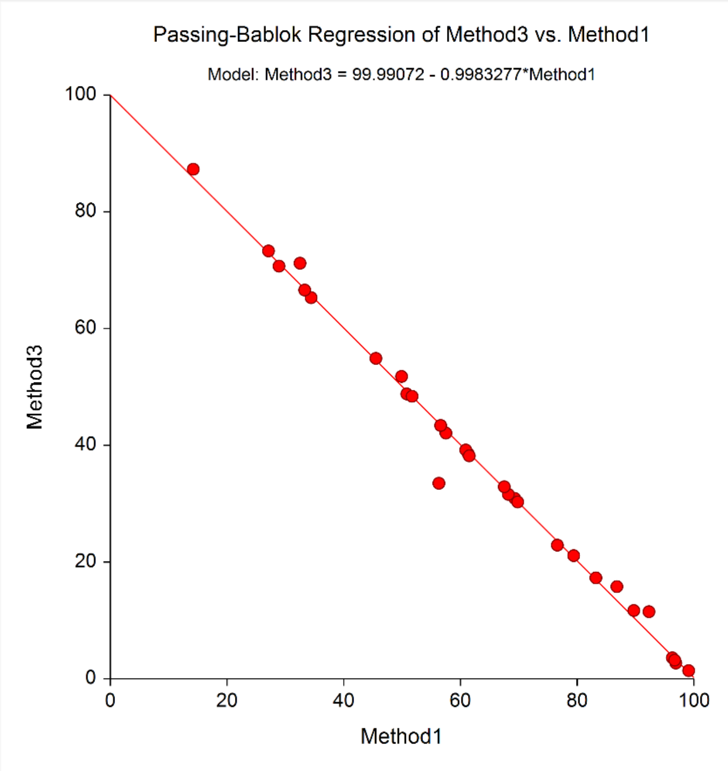
Residuals

Row	X	Y	Difference Y - X	Predicted Yhat	Residual (Y-Yhat)
1	69.3	30.9	-38.4000	30.8066	-0.0934
2	27.1	73.3	46.2000	72.9360	-0.3640
3	61.3	38.6	-22.7000	38.7932	0.1932
4	50.8	48.8	-2.0000	49.2757	0.4757
5	34.4	65.3	30.9000	65.6482	0.3482
6	92.3	11.5	-80.8000	7.8451	-3.6549
7	57.5	42.1	-15.4000	42.5869	0.4869
8	45.5	54.9	9.4000	54.5668	-0.3332

This section reports the residuals and the difference for each of the input data points.

Passing-Bablok Regression Scatter Plot

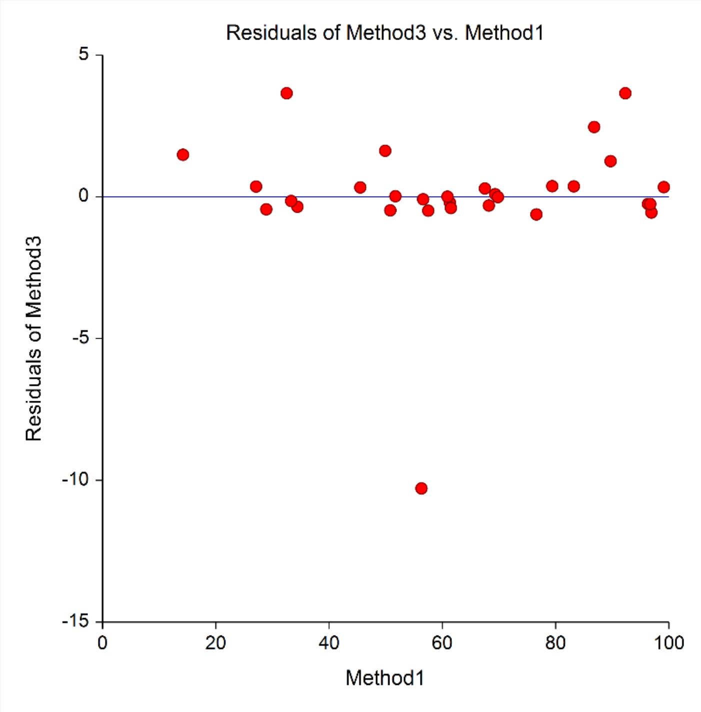
Passing-Bablok Regression Scatter Plot



This report shows the fitted Passing-Bablok regression line.

Residual Plot

Residual Plots



This plot emphasizes the deviation of the points from the regression line.