Chapter 475

Theoretical ARMA

Introduction

This procedure shows the theoretical characteristics of the autocorrelations, partial autocorrelations, and spectrum of user specified ARMA models. Unlike the other time series programs, this one does not use data. Instead, it provides a theoretical analysis of various models. It also creates simulated series from these models.

We have found this program especially useful in training and model evaluation. While you are becoming familiar with the Box-Jenkins method, this program lets you study the characteristics of a large number of models. You will be able to see the sensitivity of the autocorrelation function to changes in the number of, and values of, parameters. You will be able to generate series from known models and see how difficult it is to identify the model that they came from.

For use in theoretical model evaluation, this program factors a model written as a polynomial in the backshift operator. This will let you compare several models that each seem adequate but appear quite different. It will let you study the characteristics of various models in detail.

It is useful in model identification, because it will let you generate a catalog of possible autocorrelation patterns from known theoretical models that you can compare sample autocorrelation functions with.

All of the models treated by this program come from the general class defined by the model:

$$\phi_p(B)\Phi_P(B)X_t = \theta_q(B)\Theta_Q\left(B^s\right)a_t$$

(Refer to chapter on the Box-Jenkins method for more information on the interpretation of this equation.)

Example 1 – Using the Theoretical ARMA Procedure

This section presents an example of how to use the theoretical ARMA program.

Setup

To run this example, complete the following steps:

1 Specify the Theoretical ARMA procedure options

- Find and open the **Theoretical ARMA** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings template. To load this template, click **Open Example Template** in the Help Center or File menu.

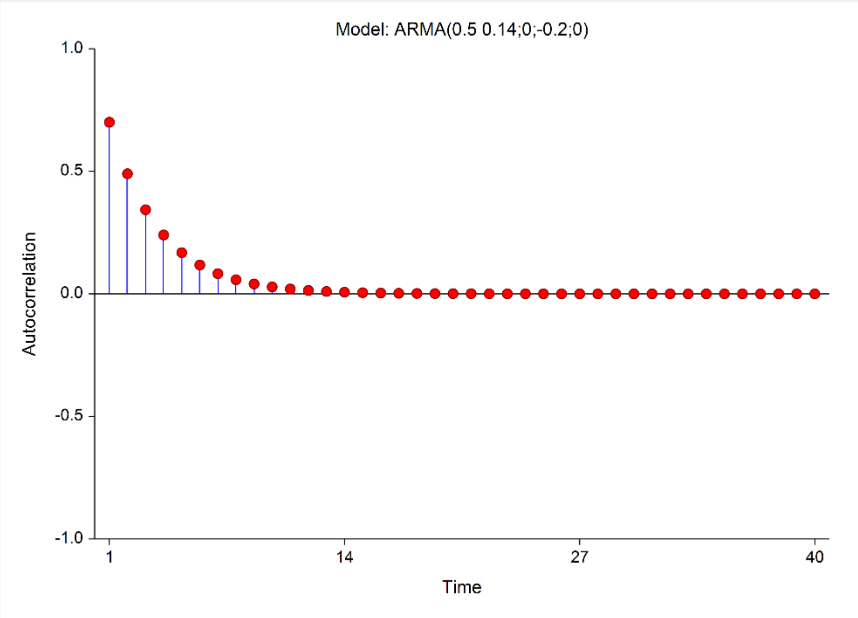
Option	Value
Data Tab	
Regular AR (Phis)	**0.5 0.14**
Regular MA (Thetas)	**-0.2**

2 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Autocorrelations Plot

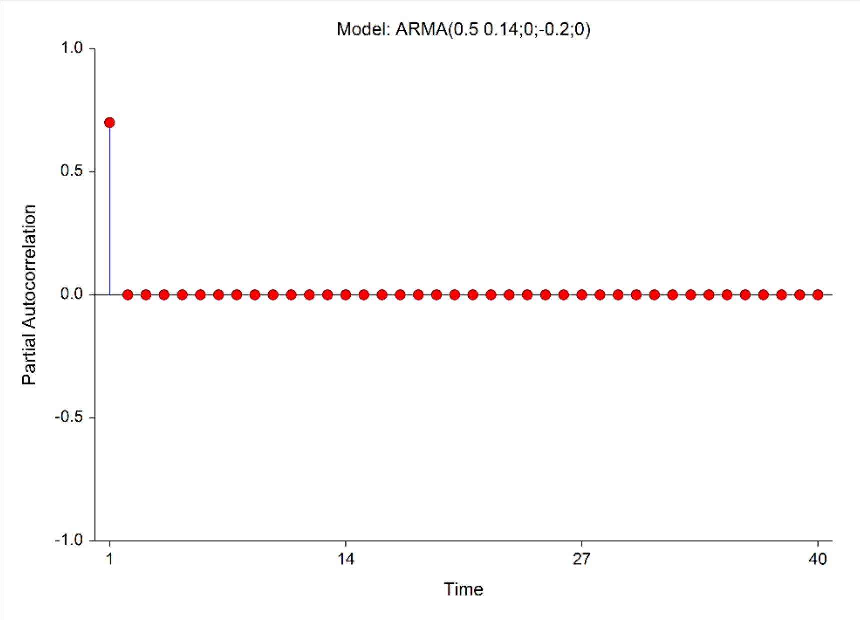
Autocorrelation Plot



This plot gives the theoretical autocorrelations.

Partial Autocorrelations Plot

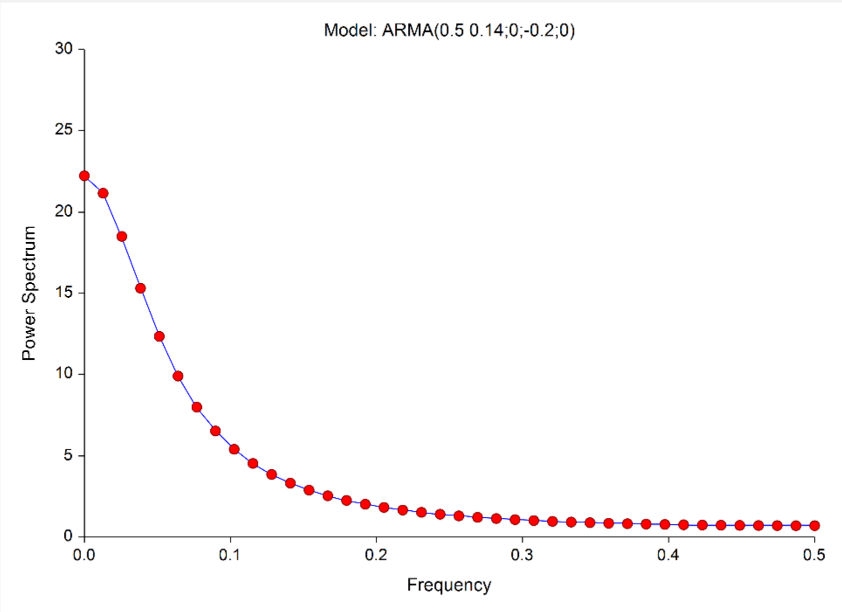
Partial Autocorrelation Plot



This plot gives the theoretical partial autocorrelations.

Power Spectrum Plot

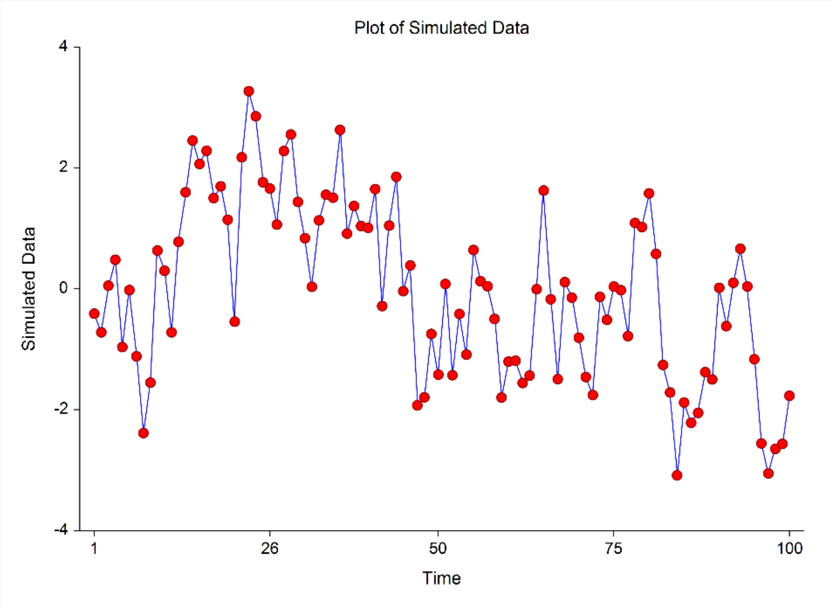
Power Spectrum Plot



This plot shows the power spectrum (see the chapter on spectral analysis).

Data Plot

Data Plot



This is a plot of a simulated data series from the model.

Autocorrelation / Power Spectrum Section

Autocorrelations / Power Spectrum Section

No.	Autocorrelations	Partial Autocorrelations	Frequency	Power Spectrum
1	0.700000	0.7	0.000000	22.2222200
2	0.490000	0.0	0.012821	21.1551200
3	0.343000	0.0	0.025641	18.4963200
4	0.240100	0.0	0.038462	15.3043900
5	0.168070	0.0	0.051282	12.3419500
6	0.117649	0.0	0.064103	9.8995800
7	0.082354	0.0	0.076923	7.9884470
8	0.057648	0.0	0.089744	6.5203100
9	0.040354	0.0	0.102564	5.3940320
10	0.028248	0.0	0.115385	4.5240170
11	0.019773	0.0	0.128205	3.8447790
12	0.013841	0.0	0.141026	3.3081010
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

This section presents the numerical values associated with the plots of the last section.

Coefficient Analysis Section

Coefficient Analysis Section

Coefficient Name	Lag	Coefficient Value	Real Root	Imaginary Root
Phi(AR)	0	1.00	-5.000000	0
Phi(AR)	1	0.50	1.428571	0
Phi(AR)	2	0.14		
Theta(MA)	0	1.00	-5.000000	0
Theta(MA)	1	-0.20		

Model is stationary and model is invertible.

This report displays an analysis of the coefficients of the model. When the model is written in terms of the backshift operator, *B*, it may be thought of as polynomials in *B*. Hence in our current example, we have two equations to study, one for the autoregressive operator and one for the moving average operator. These are:

$$\left(1-0.5B-0.14B^2\right)=0$$

and

$$\left(1+0.2B\right)=0$$

As we will show, it can be useful to find and compare the roots of these two equations, since knowledge of the roots lets us factor the equations. We see from the report that the roots of the first polynomial are 1.4286 (which is 10/7) and -5. We would like to arrange these roots so that the factors may be displayed in a standard form. To do this, we perform the following algebra on each root:

$$B = -5 \qquad\qquad B = 1.4286$$

Move the constant to the right side.

$$5 + B = 0 \qquad\qquad -1.4286 + B = 0$$

Divide by the constant.

$$1 + .2B = 0 \qquad\qquad 1 - .7B = 0$$

These are now in the special form that we can easily use them as factors. We note that the polynomial may be factored as

$$(1 -.5B -.14B2) = (1 + .2B)(1 - .7B)$$

Hence, the model (9.3) may be rewritten as

$$\left(1+0.2B\right)\left(1-0.7B\right)X_t=\left(1+0.2B\right)a_t$$

Notice that the left and right sides of this equation have $(1+0.2B)$ as a common factor. We can cancel this factor out, leaving the simpler model:

$$\left(1-0.7B\right)X_t=a_t$$

These models are equivalent. Now we can see why the partial autocorrelation plot indicated a single autoregressive parameter even though we had specified two.

A second purpose for studying these coefficients is to look for signals to difference a series. Note that if a root is approximately unity, the factor will be approximately $(1 - B)$, the difference operator. Hence, when we find roots on the autoregressive side close to one, we can simplify the model by differencing the series.

One criticism of the Box-Jenkins method is that two well-trained forecasters will most likely arrive at different models. We find that this criticism is not well-founded since often, by factoring the operator polynomials of the two models and studying their roots, we will find that the models are actually quite similar.