

## Chapter 493

# Transshipment

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## Introduction

The *transshipment* model is a special case of the *minimum cost capacitated flow* model in which there are no capacities or minimums on the arc flows. The transshipment model is similar to a transportation model, except that it allows the more realistic assumption that all nodes can transfer to and from all other nodes, no matter what their node type. Hence, it allows product to be shipped between sources and between destinations, an ability that is missing in the transportation model.

There are specialized algorithms that can be used to solve this model. However, Hillier and Lieberman (2015) show that the model can be solved using linear programming. This is the approach we have taken in NCSS.

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## Data Structure

This procedure requires a special data format that includes two regions of the spreadsheet. These are the Arc Region which define the arcs, and the Node Region which defines the net flows at the nodes.

An Arc Region, defined using spreadsheet columns, gives the arcs (links, branches, or edges) of the network. Each row in this region represents an arc. The arc definition includes the name of starting node, ending node, and incremental cost (cost of a single product). The algorithm searches for the flows in the arcs. If flows can occur in both directions within an arc, two arcs must be included in the model.

A Node Region, defined using additional spreadsheet columns, gives the net flow through each node.

## Example Model

The following dataset shows how to prepare the example on page 399 of Hillier and Lieberman (2015) for analysis. Suppose a company produces a product at two factories. It can ship product among factories, from factory to a warehouse, or from factory to two stores. Although not all factories, stores, and the warehouse are connected, many are. These connections are defined using columns Node1 and Node2. The cost of shipping a single product unit are given in the IncCost column. The supply and demand values are defined in the Node Region. The amount produced by each factory (the supply) are shown as positive net flows. Thus, Factor1 produces 50 units and Factory2 produces 40 units. The amounts needed at the two stores are 30 and 60 units. These are shown as negative numbers since these units are leaving the network. Since the warehouse does not consume any product, its net flow is zero.

Note that the sum of the net flows is exactly zero. This is a requirement of the model.

These data are stored in the dataset *Min Cost Flow*.

## Transshipment

## Min Cost Flow dataset

Node1	Node2	IncCost	Node	NetFlow
Factory1	Factory2	2	A: Factory1	50
Factory1	Warehouse	4	B: Factory2	40
Factory1	Store1	9	C: Warehouse	0
Factory2	Warehouse	3	D: Store1	-30
Warehouse	Store1	1	E: Store2	-60
Store1	Store2	3		
Store2	Store1	2		

## Linear Programming Formulation of the Minimum Cost Capacitated Flow Model

As stated earlier, we use a linear programming algorithm to solve for the maximum. Rather than present all the equations, we show how the above example is translated into a linear programming tableau. The LP tableau for the above data is

		$X_{AB}$	$X_{AC}$	$X_{AD}$	$X_{BC}$	$X_{CE}$	$X_{DE}$	$X_{ED}$	RHS
Maximize	$z =$	2	4	9	3	1	3	2	
<b>Nodes</b>									
A		1	1	1					= 50
B		-1			1				= 40
C			-1		-1	1			= 0
D				-1			1	-1	= -30
E						-1	-1	1	= -60
Capacity		10				80			

## Example 1 – Transshipment Model

### Example Model

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Store1	Store2	3		
Store2	Store1	2		

## Setup

To run this example, complete the following steps:

### 1 Open the Min Cost Flow example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Min Cost Flow** and click **OK**.

### 2 Specify the Transshipment procedure options

- Find and open the **Transshipment** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings template. To load this template, click **Open Example Template** in the Help Center or File menu.

<u>Option</u>	<u>Value</u>
<b>Specifications Tab</b>	
Arc Node 1 (Starting) Column .....	<b>Node1</b>
Arc Node 2 (Ending) Column.....	<b>Node2</b>
Arc Cost Column.....	<b>IncCost</b>
Node Name Column .....	<b>Node</b>
Node Net Flow Column .....	<b>NetFlow</b>

### 3 Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

## Transshipment

## Solution

## Solution

Arc Row	Solution Flow	IncCost	Total Cost	Node1	Node2
2	50	4	200	Factory1	Warehouse
4	40	3	120	Factory2	Warehouse
5	90	1	90	Warehouse	Store1
6	60	3	180	Store1	Store2
Total	240		590		

The solution is optimal.

This report shows the flows through each of the arcs. Arcs not listed receive a flow of zero. The flow through the network is 240 units at a cost of 590.

## Node Report

## Node Report

Type	Node	Flow In	Flow Out	Net Flow Out - In
Source	Factory1	0	50	50
Source	Factory2	0	40	40
Transshipment	Warehouse	90	90	0
Destination	Store1	90	60	-30
Destination	Store2	60	0	-60

This report shows the flows through each of the nodes. The *Flow In* is the sum of the flows of all arcs enter this node. The *Flow Out* is the sum of the flows of all arcs that leave this node.

## Model Network Arcs

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Arc Row	IncCost	Node1	Node2
1	2	Factory1 <sup>1</sup>	Factory2 <sup>1</sup>
2	4	Factory1 <sup>1</sup>	Warehouse
3	9	Factory1 <sup>1</sup>	Store1 <sup>2</sup>
4	3	Factory2 <sup>1</sup>	Warehouse
5	1	Warehouse	Store1 <sup>2</sup>
6	3	Store1 <sup>2</sup>	Store2 <sup>2</sup>
7	2	Store2 <sup>2</sup>	Store1 <sup>2</sup>

<sup>1</sup> Source (Supply) Node

<sup>2</sup> Destination (Demand) Node

This report lists all the arcs in the model as they were given on the dataset. The supply nodes are identified with a superscript of 1. The demand nodes are identified with the superscript of 2.