

## Chapter 420

# Confidence Intervals for One Mean

## Introduction

This routine calculates the sample size necessary to achieve a specified distance from the mean to the confidence limit(s) at a stated confidence level for a confidence interval about the mean when the underlying data distribution is normal.

Caution: This procedure assumes that the standard deviation of the future sample will be the same as the standard deviation that is specified. If the standard deviation to be used in the procedure is estimated from a previous sample or represents the population standard deviation, the Confidence Intervals for One Mean with Tolerance Probability procedure should be considered. That procedure controls the probability that the distance from the mean to the confidence limits will be less than or equal to the value specified.

## Technical Details

For a single mean from a normal distribution with known variance, a two-sided,  $100(1 - \alpha)\%$  confidence interval is calculated by

$$\bar{X} \pm \frac{z_{1-\alpha/2}\sigma}{\sqrt{n}}$$

A one-sided  $100(1 - \alpha)\%$  upper confidence limit is calculated by

$$\bar{X} + \frac{z_{1-\alpha}\sigma}{\sqrt{n}}$$

Similarly, the one-sided  $100(1 - \alpha)\%$  lower confidence limit is

$$\bar{X} - \frac{z_{1-\alpha}\sigma}{\sqrt{n}}$$

For a single mean from a normal distribution with unknown variance, a two-sided,  $100(1 - \alpha)\%$  confidence interval is calculated by

$$\bar{X} \pm \frac{t_{1-\alpha/2, n-1}\hat{\sigma}}{\sqrt{n}}$$

A one-sided  $100(1 - \alpha)\%$  upper confidence limit is calculated by

$$\bar{X} + \frac{t_{1-\alpha, n-1}\hat{\sigma}}{\sqrt{n}}$$

## Confidence Intervals for One Mean

Similarly, the one-sided  $100(1 - \alpha)\%$  lower confidence limit is

$$\bar{X} - \frac{t_{1-\alpha, n-1} \hat{\sigma}}{\sqrt{n}}$$

Each confidence interval is calculated using an estimate of the mean plus and/or minus a quantity that represents the distance from the mean to the edge of the interval. For two-sided confidence intervals, this distance is sometimes called the precision, margin of error, or half-width. We will label this distance,  $D$ .

The basic equation for determining sample size when  $D$  has been specified is

$$D = \frac{z_{1-\alpha/2} \sigma}{\sqrt{n}}$$

when the standard deviation is known, and

$$D = \frac{t_{1-\alpha/2, n-1} \hat{\sigma}}{\sqrt{n}}$$

when the standard deviation is unknown. These equations can be solved for any of the unknown quantities in terms of the others. The value  $\alpha/2$  is replaced by  $\alpha$  when a one-sided interval is used.

## Finite Population Size

The above calculations assume that samples are being drawn from a large (infinite) population. When the population is of finite size ( $N$ ), an adjustment must be made. The adjustment reduces the standard deviation as follows:

$$\sigma_{finite} = \sigma \sqrt{\left(1 - \frac{n}{N}\right)}$$

This new standard deviation replaces the regular standard deviation in the above formulas.

## Confidence Level

The confidence level,  $1 - \alpha$ , has the following interpretation. If thousands of samples of  $n$  items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population mean is  $1 - \alpha$ .

## Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

## Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

### Solve For

#### Solve For

This option specifies the parameter to be solved for from the other parameters.

## Confidence Intervals for One Mean

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### One-Sided or Two-Sided Interval

#### Interval Type

Specify whether the interval to be used will be a one-sided or a two-sided confidence interval.

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### Population

#### Population Size

This is the number of individuals in the population. Usually, you assume that samples are drawn from a very large (infinite) population. Occasionally, however, situations arise in which the population of interest is of limited size. In these cases, appropriate adjustments must be made. This option sets the population size.

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### Confidence

#### Confidence Level

The confidence level,  $1 - \alpha$ , has the following interpretation. If thousands of samples of  $n$  items are drawn from a population using simple random sampling and a confidence interval is calculated for each sample, the proportion of those intervals that will include the true population mean is  $1 - \alpha$ .

Often, the values 0.95 or 0.99 are used. You can enter single values or a range of values such as *0.90, 0.95* or *0.90 to 0.99 by 0.01*.

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### Sample Size

#### N (Sample Size)

Enter one or more values for the sample size. This is the number of individuals selected at random from the population to be in the study.

You can enter a single value or a range of values.

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### Precision

#### Distance from Mean to Limit(s)

This is the distance from the confidence limit(s) to the mean. For two-sided intervals, it is also known as the precision, half-width, or margin of error.

You can enter a single value or a list of values. The value(s) must be greater than zero.

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### Standard Deviation

#### S (Standard Deviation)

Enter a value (or range of values) for the standard deviation. Roughly speaking, this value estimates the average absolute difference between each individual and every other individual. You can use the results of a pilot study, a previous study, or a ball park estimate based on the range (e.g.,  $\text{Range}/4$ ) to estimate this parameter.

#### Know Standard Deviation

Check this box when you want to base your results on the normal distribution. When the box is not checked, calculations are based on the t-distribution. The difference between the two distributions is negligible when the sample sizes are large ( $>50$ ).

## Confidence Intervals for One Mean

## Example 1 – Calculating Sample Size

Suppose a study is planned in which the researcher wishes to construct a two-sided 95% confidence interval for the mean such that the width of the interval is no wider than 14 units. The confidence level is set at 0.95, but 0.99 is included for comparative purposes. The standard deviation estimate, based on the range of data values, is 28. Instead of examining only the interval half-width of 7, a series of half-widths from 5 to 9 will also be considered. The goal is to determine the necessary sample size.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for One Mean** procedure window by expanding **Means**, then **One Mean**, then clicking on **Confidence Interval**, and then clicking on **Confidence Intervals for One Mean**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size</b>
Interval Type .....	<b>Two-Sided</b>
Population Size.....	<b>Infinite</b>
Confidence Level.....	<b>0.95 0.99</b>
Distance from Mean to Limit(s).....	<b>5 to 9 by 1</b>
S (Standard Deviation) .....	<b>28</b>
Known Standard Deviation .....	<b>Not Checked</b>

### Annotated Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Results

#### Numeric Results for Two-Sided Confidence Intervals with Unknown Standard Deviation

Confidence Level	Sample Size (N)	Target Distance from Mean to Limits	Actual Distance from Mean to Limits	Standard Deviation (S)
0.950	123	5.000	4.998	28.000
0.990	212	5.000	4.999	28.000
0.950	87	6.000	5.968	28.000
0.990	149	6.000	5.986	28.000
0.950	64	7.000	6.994	28.000
0.990	110	7.000	6.999	28.000
0.950	50	8.000	7.958	28.000
0.990	86	8.000	7.956	28.000
0.950	40	9.000	8.955	28.000
0.990	69	9.000	8.933	28.000

#### References

Hahn, G. J. and Meeker, W.Q. 1991. Statistical Intervals. John Wiley & Sons. New York.

## Confidence Intervals for One Mean

### Report Definitions

Confidence level is the proportion of confidence intervals (constructed with this same confidence level, sample size, etc.) that would contain the population mean.

N is the size of the sample drawn from the population.

Distance from Mean to Limit is the distance from the confidence limit(s) to the mean. For two-sided intervals, it is also known as the precision, half-width, or margin of error.

Target Distance from Mean to Limit is the value of the distance that is entered into the procedure.

Actual Distance from Mean to Limit is the value of the distance that is obtained from the procedure.

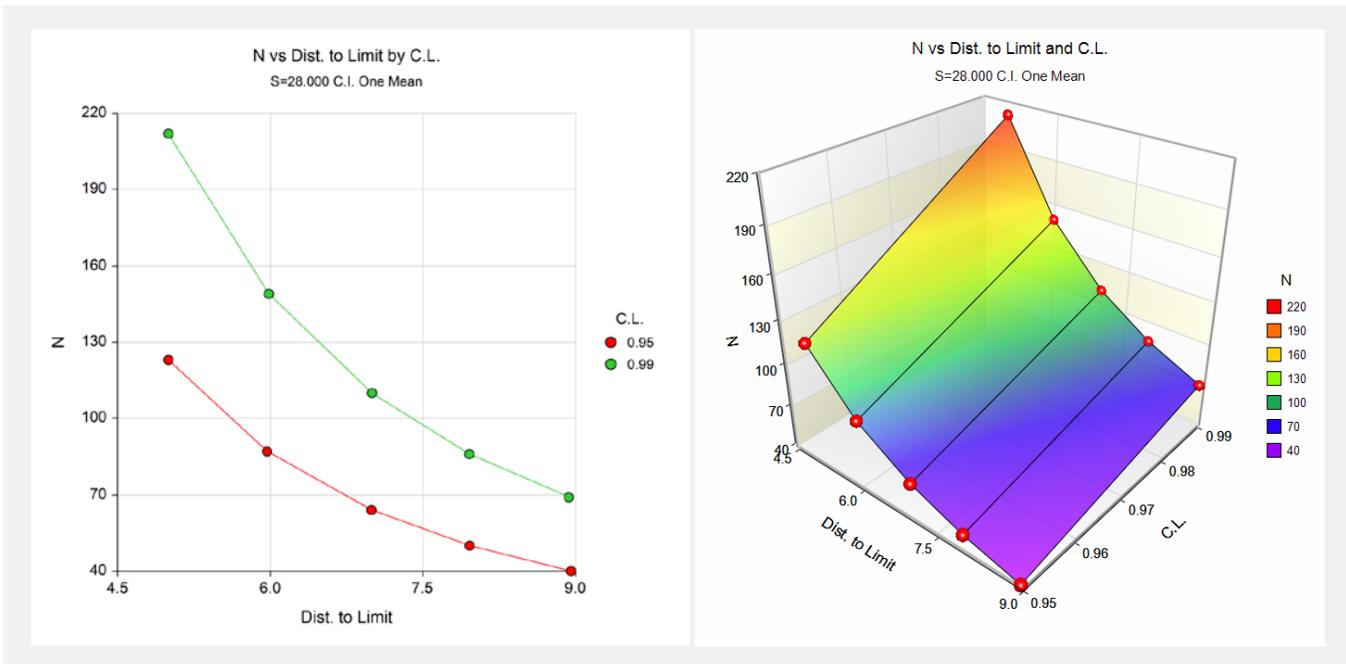
The standard deviation of the population measures the variability in the population.

### Summary Statements

A sample size of 123 produces a two-sided 95% confidence interval with a distance from the mean to the limits that is equal to 4.998 when the estimated standard deviation is 28.000.

This report shows the calculated sample size for each of the scenarios.

### Chart Section



These plots show the sample size versus the distance from the mean to the limits (precision) for the two confidence levels.

## Confidence Intervals for One Mean

**Example 2 – Validation using Moore and McCabe**

Moore and McCabe (1999) page 443 give an example of a sample size calculation for a confidence interval on the mean when the confidence coefficient is 95%, the standard deviation is known to be 3, and the margin of error is 2. The necessary sample size is 9.

**Setup**

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for One Mean** procedure window by expanding **Means**, then **One Mean**, then clicking on **Confidence Interval**, and then clicking on **Confidence Intervals for One Mean**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size</b>
Interval Type .....	<b>Two-Sided</b>
Population Size .....	<b>Infinite</b>
Confidence Level .....	<b>0.95</b>
Distance from Mean to Limit(s) .....	<b>2</b>
S (Standard Deviation) .....	<b>3</b>
Known Standard Deviation .....	<b>Checked</b>

**Output**

Click the Calculate button to perform the calculations and generate the following output.

**Numeric Results**

Confidence Level	Sample Size (N)	Target Distance from Mean to Limits	Actual Distance from Mean to Limits	Standard Deviation (S)
0.95	9	2.000	1.960	3.000

PASS also calculated the necessary sample size to be 9.

## Example 3 – Validation using Ostle and Malone

Ostle and Malone (1988) page 536 give an example of a sample size calculation for a confidence interval on the mean when the confidence coefficient is 95%, the standard deviation is known to be 7, and the margin of error is 5. The necessary sample size is 8.

### Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Confidence Intervals for One Mean** procedure window by expanding **Means**, then **One Mean**, then clicking on **Confidence Interval**, and then clicking on **Confidence Intervals for One Mean**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
<b>Design Tab</b>	
Solve For .....	<b>Sample Size</b>
Interval Type .....	<b>Two-Sided</b>
Population Size .....	<b>Infinite</b>
Confidence Level .....	<b>0.95</b>
Distance from Mean to Limit(s) .....	<b>5</b>
S (Standard Deviation) .....	<b>7</b>
Known Standard Deviation .....	<b>Checked</b>

### Output

Click the Calculate button to perform the calculations and generate the following output.

#### Numeric Results

Confidence Level	Sample Size (N)	Target Distance from Mean to Limits	Actual Distance from Mean to Limits	Standard Deviation (S)
0.95	8	5.000	4.851	7.000

PASS also calculated the necessary sample size to be 8.